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Remote Preparation of an Arbitrary Two-Qubit State with Three-Party

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Abstract In this paper, we propose a scheme to remotely prepare an arbitrary two-qubit state from one sender to either of two receivers. Two cases of the prepared quantum state, an arbitrary two-qubit state with real coefficients and complex coefficients, are considered. Firstly, one single EPR pair and a GHZ state are used as the quantum channel. Then the present scheme is extended to some partially entangled states as the quantum channel. To design these schemes, some useful and general measurement bases have been constructed. The successful probability and classical communication cost of these schemes are also calculated to weigh the efficiency and cost.

Keywords Remote state preparation · EPR pair · GHZ state · Two-qubit state

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1 Introduction

Secure transmission of a quantum state from one place to another is an elementary action for different tasks in quantum network communication and quantum distributed computation. However, a quantum state can not be sent directly because it can be replaced by any enemy. To achieve secure transmission, Bennett et al. [1] introduced quantum teleportation to transmit an unknown state by using both quantum and classical channels. When the prepared quantum state is known to the sender, Lo [2] presented a new method called remote state preparation (RSP) to prepare a pure quantum state. This scheme can be done via the same quantum channel as in teleportation but with simpler measurements and less classical communication cost (CCC). Bennett et al. [3] studied the trade-off between quantum resources and classical communication in RSP. Since then, researchers have made various generalizations of theoretical RSP schemes, including low-entanglement RSP [4], optimal RSP [5], generalized RSP [6], oblivious RSP [7], faithful RSP [8] and continuous variable RSP in phase space [9], etc. Moreover, RSP schemes have been implemented experimentally [10, 11], etc.

The RSP with multi-party plays an important role in the general quantum network communication and quantum distributed computation. However, compared with the classical RSP with two-party, few proposals have been made for remotely preparing a quantum state with multi-party. Joint preparation of a quantum state with three-party has been considered [12–15]. Another remote preparation of an entangled two-qubit state from a sender to either of two receivers has been proposed [16], then generalized to an entangled multi-qubit state [17]. In this paper, based on some ideas in [16, 17], one protocol is proposed to remotely prepare an arbitrary two-qubit state from a sender to either of two receivers. This scheme can also be viewed as the quantum state sharing scheme [18–22] or the controlled RSP [14, 23].

Moreover, all classical RSP protocols in quantum information processing require a known initial quantum state for the sender, and typically have optimal results for maximally entangled initial states. However, decoherence and dissipation may cause the states to become less entangled. Therefore, on the one hand, it is necessary and meaningful to investigate the RSP of a general multi-qubit state. On the other hand, it is also important to design RSP by using the various quantum resources, such as maximal entanglement or partial entanglement as the quantum channel. Based on these features, our protocols use the combination of an EPR pair [24] and a GHZ state [25] or their partial entanglements as the quantum channel. To design these schemes, we construct some general and useful measurement bases for the sender. With the aid of these measurement bases the sender can remotely prepare a two-qubit state with real coefficients with the unit probability, and prepare a two-qubit state with complex coefficients with the probability 25%. In order to weigh the classical resources required, we calculate the total CCC of all the schemes.

The rest parts of paper are organized as follows. In Sect. 2, we propose a scheme to remotely prepare an arbitrary two-qubit state with real coefficients from a sender to either of two receivers. A general and useful measurement basis has been constructed for the sender. It is shown that such RSP scheme possesses the successful probability 100%. This scheme is extended to the general scheme with complex coefficients, shown in Sect. 3. Comparing with the scheme in Sect. 2, the general RSP scheme possesses the successful probability 25%. These schemes take use of an EPR pair and a GHZ state as the quantum channel. When the partially entangled states are used as the quantum channel, the sender can take the local filtering to obtain the maximally entangled quantum channel, its details are shown in Sect. 4. Of course, another equivalent scheme is also presented in this section. In Sect. 5, we calculate the CCC which requires only 3 cbits and 1.25 cbits for the present schemes in Sect. 2 and Sect. 3, respectively. Some discussions and conclusions are given in Sect. 6.

2 Remote Preparation of an Arbitrary Two-Qubit State with Real Coefficients

Suppose that the sender Alice wants to help either of two receivers Bob and Charlie remotely prepare an arbitrary two-qubit state

$$|\phi\rangle = a_1|00\rangle + a_2|01\rangle + a_3|10\rangle + a_4|11\rangle, \tag{1}$$

where a_i, $1 \le i \le 4$, are real numbers with $\sum_{i=1}^{4} a_i^2 = 1$. Alice knows about a_i completely, but both Bob and Charlie do not know them at all. We also suppose that the quantum channel is composed of an EPR pair $|\Psi\rangle_{12}$ and a GHZ state $|\Psi\rangle_{345}$ as follows:

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{12}, \tag{2}$$

$$|\Psi\rangle_{345} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{345}. \tag{3}$$

Here, the particles $(1, 3)$ are in the possession of Alice, the particle 2 only belongs to one of receivers, Bob or Charlie, while the particles 4 and 5 belong to Bob and Charlie, respectively. Figure 1 shows the schematic diagram for preparing an arbitrary two-qubit state with three parties, without loss of generality, where Bob is assumed to be the receiver with the extra particle 2.

The following RSP can be divided into three steps.

Step 1. Sender performs some operations on particles $(1, 3)$.

To help Bob or Charlie remotely prepare the state shown in (1), we find a useful unitary matrix U, where

$$U = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_2 & -a_1 & a_4 & -a_3 \\ a_3 & -a_4 & -a_1 & a_2 \\ -a_4 & -a_3 & a_2 & a_1 \end{pmatrix}. \tag{4}$$

Then a general measurement basis $\{|\psi_{00}\rangle, |\psi_{01}\rangle, |\psi_{10}\rangle, |\psi_{11}\rangle\}$ can be constructed by U. In detail, this basis is related to the computation basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ by U^{T} (transpose

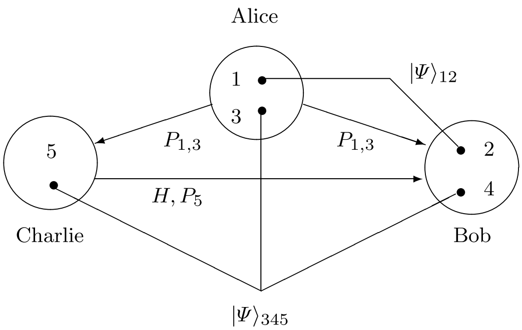

Fig. 1 A sketch of the RSP of an arbitrary two-qubit state with three-party. An EPR pair $|\Psi\rangle_{12}$ and a GHZ state $|\Psi\rangle_{345}$ are used as the quantum channel. $P_{1,3}$ stands for a projective measurement on particles $(1, 3)$, which is performed by Alice. H and P_5 stand for Hadamard operation and a projective measurement on particle 5, respectively, which are performed by Charlie. In this figure, a qubit is represented by a dot, entangled qubits are connected by *solid lines* and a classical communication from one site to another is represented by a vector

of U):

$$\begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & a_3 & -a_4 \\ a_2 & -a_1 & -a_4 & -a_3 \\ a_3 & a_4 & -a_1 & a_2 \\ a_4 & -a_3 & a_2 & a_1 \end{pmatrix} \begin{pmatrix} |\psi_{00}\rangle \\ |\psi_{01}\rangle \\ |\psi_{10}\rangle \\ |\psi_{11}\rangle \end{pmatrix}. \tag{5}$$

With this basis the quantum channel can be rewritten into

$$\begin{aligned} |\Psi\rangle_{12} \otimes |\Psi\rangle_{345} = {} & \frac{1}{2}|\psi_{00}\rangle_{13}(a_1|000\rangle + a_2|011\rangle + a_3|100\rangle + a_4|111\rangle)_{245} \\ & + \frac{1}{2}|\psi_{01}\rangle_{13}(a_2|000\rangle - a_1|011\rangle + a_4|100\rangle - a_3|111\rangle)_{245} \\ & + \frac{1}{2}|\psi_{10}\rangle_{13}(a_3|000\rangle - a_4|011\rangle - a_1|100\rangle + a_2|111\rangle)_{245} \\ & + \frac{1}{2}|\psi_{11}\rangle_{13}(-a_4|000\rangle - a_3|011\rangle + a_2|100\rangle + a_1|111\rangle)_{245}. \end{aligned} \tag{6}$$

Now, Alice performs a projective measurement $P_{1,3}$ on particles $(1, 3)$ under the general basis $\{|\psi_{00}\rangle, |\psi_{01}\rangle, |\psi_{10}\rangle, |\psi_{11}\rangle\}$ and broadcasts her measurement outcome via classical communication to Bob and Charlie. For example, if the von Neumann measurement result is $|\psi_{10}\rangle_{13}$, Alice broadcasts 10 (a coding form of $|\psi_{10}\rangle_{13}$) to Bob and Charlie, the state of particles $(2, 4, 5)$ collapses into (up to a global phase factor)

$$(a_3|000\rangle - a_4|011\rangle - a_1|100\rangle + a_2|111\rangle)_{245}. \tag{7}$$

Step 2. Assistant performs some operations on particle 5.

If Bob wants to get the state in (1), particle 2 belongs to him. Assistant Charlie performs the Hadamard operation H on particle 5, where

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \qquad H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle), \tag{8}$$

then the state shown in (7) is transformed to

$$\begin{aligned} & \frac{1}{\sqrt{2}}(a_3|00\rangle - a_4|01\rangle - a_1|10\rangle + a_2|11\rangle)_{24}|0\rangle_5 \\ & + \frac{1}{\sqrt{2}}(a_3|00\rangle + a_4|01\rangle - a_1|10\rangle - a_2|11\rangle)_{24}|1\rangle_5. \end{aligned} \tag{9}$$

Charlie performs a projective measurement P_5 on particle 5 under the basis $\{|0\rangle_5, |1\rangle_5\}$. If Charlie obtains $|1\rangle_5$, then the state shown in (9) will collapse into (up to a global phase factor)

$$(a_3|00\rangle + a_4|01\rangle - a_1|10\rangle - a_2|11\rangle)_{24}. \tag{10}$$

After the measurement, Charlie sends her measurement outcome to Bob via classical communication.

Step 3. Receiver performs some recovery operations.

Depending on Alice's and Charlie's measurement outcomes, Bob carries out the unitary transformation $(\sigma_x\sigma_z)_2 \otimes I_4$ on particles $(2, 4)$ of the state shown in (10) and obtains

$$(a_1|00\rangle + a_2|01\rangle + a_3|10\rangle + a_4|11\rangle)_{24}, \tag{11}$$

Table 1 The recovery transformations conditional on the measurement outcomes of Alice and Charlie are performed on particles $(2, 4)$ by Bob. I is an identity operator, σ_x and σ_z are Pauli matrices

	$\|0\rangle_5$	$\|1\rangle_5$
$\|\psi_{00}\rangle_{13}$	$I_2 \otimes I_4$	$I_2 \otimes (\sigma_z)_4$
$\|\psi_{01}\rangle_{13}$	$I_2 \otimes (\sigma_x\sigma_z)_4$	$I_2 \otimes (\sigma_x)_4$
$\|\psi_{10}\rangle_{13}$	$(\sigma_x\sigma_z)_2 \otimes (\sigma_z)_4$	$(\sigma_x\sigma_z)_2 \otimes I_4$
$\|\psi_{11}\rangle_{13}$	$(-\sigma_x\sigma_z)_2 \otimes (\sigma_x)_4$	$(-\sigma_x\sigma_z)_2 \otimes (\sigma_x\sigma_z)_4$

where

$$(\sigma_x\sigma_z)_2 \otimes I_4 = (|1\rangle\langle 0| - |0\rangle\langle 1|)_2 \otimes (|0\rangle\langle 0| + |1\rangle\langle 1|)_4 \tag{12}$$

and Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{13}$$

Thus Bob obtains the prepared state shown in (1).

With the similar method above, if Charlie's measurement result is $|0\rangle_5$, Bob obtains the quantum state $(a_3|00\rangle - a_4|01\rangle - a_1|10\rangle + a_2|11\rangle)_{24}$. He can obtain $|\phi\rangle$ in (1) by performing the unitary transformation $(\sigma_x\sigma_z)_2 \otimes (\sigma_z)_4$ on particles $(2, 4)$. So, from (9) the successful probability of RSP is $\frac{1}{4}$.

Similarly, for each Alice's measurement result of $\{|\psi_{00}\rangle_{13}, |\psi_{01}\rangle_{13}, |\psi_{11}\rangle_{13}\}$, Bob can recover the prepared state shown in (1) with the successful probability $\frac{1}{4}$. Therefore, the total successful probability of this RSP is $4 \times \frac{1}{4} = 1$. The recovery unitary transformations applied by Bob on particles $(2, 4)$ are shown in Table 1.

3 Remote Preparation of an Arbitrary Two-Qubit State with Complex Coefficients

In this section, we extend the protocol in Sect. 2 to the general case when the coefficients a_i are all complex. The quantum channel is composed of the states shown in (2) and (3). In this case, instead of (5), Alice has to use another mutually orthogonal basis. To design this protocol, we find another unitary matrix V by performing the Gram-Schmidt orthogonal procedure on linearly independent vectors set $\{(a_1^*, a_2^*, a_3^*, a_4^*), (a_2, -a_1, a_4, -a_3), (a_3, a_4, -a_1, a_2), (a_4, -a_3, a_2, a_1)\}$, without loss of generality, denotes

$$V = \begin{pmatrix} a_1^* & a_2^* & a_3^* & a_4^* \\ a_2 & -a_1 & a_4 & -a_3 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \end{pmatrix}. \tag{14}$$

Then Alice can take use of the basis $\{|\hat{\psi}_{00}\rangle, |\hat{\psi}_{01}\rangle, |\hat{\psi}_{10}\rangle, |\hat{\psi}_{11}\rangle\}$, which is related to the computation basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ by the conjugate transpose of V:

$$\begin{pmatrix} |\hat{\psi}_{00}\rangle & |\hat{\psi}_{01}\rangle & |\hat{\psi}_{10}\rangle & |\hat{\psi}_{11}\rangle \end{pmatrix}^{\mathrm{T}} = V \begin{pmatrix} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{pmatrix}^{\mathrm{T}}, \tag{15}$$

where the superscript T denotes the transpose of a vector or matrix. Of course, this basis is not unique and it can be obtained from other linearly independent vectors sets. Now, with the basis $\{|\hat{\psi}_{00}\rangle, |\hat{\psi}_{01}\rangle, |\hat{\psi}_{10}\rangle, |\hat{\psi}_{11}\rangle\}$ the quantum channel $|\Psi\rangle_{12} \otimes |\Psi\rangle_{345}$ can be rewritten as

$$\begin{aligned}|\Psi\rangle_{12} \otimes |\Psi\rangle_{345} = {} & \frac{1}{2}|\hat{\psi}_{00}\rangle_{13}(a_1|000\rangle + a_2|011\rangle + a_3|100\rangle + a_4|111\rangle)_{245} \\ & + \frac{1}{2}|\hat{\psi}_{01}\rangle_{13}(a_2^*|000\rangle - a_1^*|011\rangle + a_4^*|100\rangle - a_3^*|111\rangle)_{245} \\ & + \frac{1}{2}|\hat{\psi}_{10}\rangle_{13}(b_1^*|000\rangle + b_2^*|011\rangle + b_3^*|100\rangle + b_4^*|111\rangle)_{245} \\ & + \frac{1}{2}|\hat{\psi}_{11}\rangle_{13}(c_1^*|000\rangle + c_2^*|011\rangle + c_3^*|100\rangle + c_4^*|111\rangle)_{245}.\end{aligned} \tag{16}$$

Different from the present scheme in Sect. 2, Alice measures her qubits under the basis $\{|\hat{\psi}_{00}\rangle_{13}, |\hat{\psi}_{01}\rangle_{13}, |\hat{\psi}_{10}\rangle_{13}, |\hat{\psi}_{11}\rangle_{13}\}$ and broadcasts the measurement result to Bob and Charlie. If Alice's measurement result is $|\hat{\psi}_{00}\rangle_{13}$, from (16) the state of particles $(2, 4, 5)$ collapses into (up to a global phase factor)

$$(a_1|000\rangle + a_2|011\rangle + a_3|100\rangle + a_4|111\rangle)_{245} \tag{17}$$

which is same as Alice's measurement result $|\psi_{00}\rangle_{13}$ in Sect. 2. So with the same method in Sect. 2, Bob can recover the prepared state $|\phi\rangle$ shown in (1) with the probability $\frac{1}{4}$. The corresponding recovery transformations conditional on the measurement outcomes of Alice and Charlie are shown in Table 1. As for Alice's measurement result $\{|\hat{\psi}_{01}\rangle_{13}, |\hat{\psi}_{10}\rangle_{13}, |\hat{\psi}_{11}\rangle_{13}\}$, from (16) the conjugates of coefficients $a_i (1 \le i \le 4)$ are always included in the collapsed states of particles $(2, 4, 5)$. So Charlie can not help Bob to prepare the state shown in (1) due to no knowledge of coefficients $a_i (i = 1, \dots, 4)$. Therefore, a successful RSP of the state shown in (1) with complex coefficients a_i is achieved only for the measurement outcome $|\hat{\psi}_{00}\rangle_{13}$, with the probability $P = \frac{1}{4}$.

4 Partially Entangled Quantum Channel

In this section, we extend the present schemes to the general case where the non-maximally entangled states are used as the quantum channel. In detail, the quantum channel is composed of a partially entangled EPR pair $|\hat{\Psi}\rangle_{12}$ and a partially entangled GHZ state $|\hat{\Psi}\rangle_{345}$ as follows:

$$|\hat{\Psi}\rangle_{12} = (a|00\rangle + b|11\rangle)_{12}, \tag{18}$$

$$|\hat{\Psi}\rangle_{345} = (c|000\rangle + d|111\rangle)_{345}, \tag{19}$$

where $|a|^2 + |b|^2 = |c|^2 + |d|^2 = 1$. Without loss of generality, assume $|a| \le |b|$ and $|c| \le |d|$. The coefficients a, b, c, d are known to Alice, Charlie and Bob. In the follow, we can design two preparation schemes with same successful probability.

On the one hand, Alice, Charlie or Bob first converts the partially entangled quantum channel to the maximally entangled quantum channel with a certain probability by the normal local filtering [26]. Then followed the present schemes via the maximally entangled quantum channel in Sect. 2 or Sect. 3, Alice can complete preparing an arbitrary two-qubit state to either of two receivers. In detail, take $|\hat{\Psi}\rangle_{12}$ shown in (18) and the sender Alice as an example, by introducing an auxiliary two-level particle with initial state $|0\rangle_A$

into $|\hat{\Psi}\rangle_{12}$ and performing a unitary transformation W on particles $(1, A)$ under the basis $\{|0\rangle_1|0\rangle_A, |1\rangle_1|0\rangle_A, |0\rangle_1|1\rangle_A, |1\rangle_1|1\rangle_A\}$, Alice obtains

$$|\hat{\Psi}\rangle_{12}\otimes|0\rangle_A = a(|00\rangle+|11\rangle)_{12}\otimes|0\rangle_A + b\sqrt{1-\left|\frac{a}{b}\right|^2}\,|11\rangle_{12}\otimes|1\rangle_A, \tag{20}$$

where

$$W=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \frac{a}{b} & 0 & \sqrt{1-|\frac{a}{b}|^2}\\ 0 & 0 & -1 & 0\\ 0 & \sqrt{1-|\frac{a}{b}|^2} & 0 & -\frac{a^*}{b^*}\end{pmatrix}. \tag{21}$$

Then, Alice takes a projective measurement on auxiliary particle A under the basis $\{|0\rangle, |1\rangle|\}$. If the measurement result is $|0\rangle_A$, Alice obtain the maximally entangled state $\frac{\sqrt{2}}{2}(|00\rangle+|11\rangle)_{12}$ with the probability $2|a|^2$, i.e., the local filtering succeeds. If measurement result is $|1\rangle_A$, the local filtering fails. Similarly, by the local filtering Alice can get the maximally entangled state $\frac{\sqrt{2}}{2}(|000\rangle+|111\rangle)_{345}$ from $|\hat{\Psi}\rangle_{345}$ shown in (19) with the probability $2|c|^2$. Thus Alice can change the quantum channel composed of the states shown in (18) and (19) into

$$\frac{1}{2}(|00\rangle+|11\rangle)_{12}\otimes(|000\rangle+|111\rangle)_{345} \tag{22}$$

with the probability $4|ac|^2$ by the local filtering, which is same as the quantum channel used in Sects. 2 and 3. Finally, followed the protocols in Sect. 2 and Sect. 3, Alice can complete the preparation for Bob or Charlie with the successful probabilities $4|ac|^2$ and $4|ac|^2\times\frac{1}{4}=|ac|^2$, respectively.

On the other hand, Alice and Charlie first follow the scheme in Sect. 2 or Sect. 3, and then Bob performs the recovery operations and the local filtering. Take the scheme in Sect. 2 as an example, instead of (6) the quantum channel $|\hat{\Psi}\rangle_{12}\otimes|\hat{\Psi}\rangle_{345}$ can be rewritten as

$$\begin{aligned}|\hat{\Psi}\rangle_{12}\otimes|\hat{\Psi}\rangle_{345} &= \frac{1}{2}|\psi_{00}\rangle_{13}(a_1ac|000\rangle+a_2ad|011\rangle+a_3bc|100\rangle+a_4bd|111\rangle)_{245}\\ &\quad+\frac{1}{2}|\psi_{01}\rangle_{13}(a_2ac|000\rangle-a_1ad|011\rangle+a_4bc|100\rangle-a_3bd|111\rangle)_{245}\\ &\quad+\frac{1}{2}|\psi_{10}\rangle_{13}(a_3ac|000\rangle-a_4ad|011\rangle-a_1bc|100\rangle+a_2bd|111\rangle)_{245}\\ &\quad+\frac{1}{2}|\psi_{11}\rangle_{13}(-a_4ac|000\rangle-a_3ad|011\rangle+a_2bc|100\rangle+a_1bd|111\rangle)_{245}.\end{aligned} \tag{23}$$

Followed the steps 1 and 2 in Sect. 2 for Alice and Charlie, the receiver Bob obtains

$$(aca_3|00\rangle+ada_4|01\rangle-bca_1|10\rangle-bda_2|11\rangle)_{24} \tag{24}$$

instead of the state shown in (10) for Alice's measurement outcome $|\psi_{10}\rangle_{13}$ and Charlie's measurement outcome $|1\rangle_5$. Then by introducing an auxiliary two-level particle A with initial state $|0\rangle_A$ into the state shown in (24) and performing a unitary transformation $\tilde{W}$ on particles $(2, 4, A)$ under the basis $\{|00\rangle_{24}|0\rangle_A, |01\rangle_{24}|0\rangle_A, |10\rangle_{24}|0\rangle_A, |11\rangle_{24}|0\rangle_A, |00\rangle_{24}|1\rangle_A, |01\rangle_{24}|1\rangle_A, |10\rangle_{24}|1\rangle_A, |11\rangle_{24}|1\rangle_A\}$, Bob can obtain

$$
\begin{aligned}
&ac(a_3|00\rangle + a_4|01\rangle - a_1|10\rangle - a_2|11\rangle)_{24} \otimes |0\rangle_A \\
&\quad + \left(ada_4\sqrt{1-\left|\frac{c}{d}\right|^2}|01\rangle - bca_1\sqrt{1-\left|\frac{a}{b}\right|^2}|10\rangle - bda_2\sqrt{1-\left|\frac{ac}{bd}\right|^2}|11\rangle\right)_{24} \otimes |1\rangle_A,
\end{aligned}
\tag{25}
$$

where

$$
\tilde{W} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{c}{d} & 0 & 0 & 0 & \sqrt{1-|\frac{c}{d}|^2} & 0 & 0 \\
0 & 0 & \frac{a}{b} & 0 & 0 & 0 & \sqrt{1-|\frac{a}{b}|^2} & 0 \\
0 & 0 & 0 & \frac{ac}{bd} & 0 & 0 & 0 & \sqrt{1-|\frac{ac}{bd}|^2} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & \sqrt{1-|\frac{c}{d}|^2} & 0 & 0 & 0 & -\frac{c^*}{d^*} & 0 & 0 \\
0 & 0 & \sqrt{1-|\frac{a}{b}|^2} & 0 & 0 & 0 & -\frac{a^*}{b^*} & 0 \\
0 & 0 & 0 & \sqrt{1-|\frac{ac}{bd}|^2} & 0 & 0 & 0 & -\frac{a^*c^*}{b^*d^*}
\end{pmatrix}.
\tag{26}
$$

So by measuring auxiliary particle A under the basis $\{|0\rangle_A, |1\rangle_A\}$, Bob obtains

$$
(a_3|00\rangle + a_4|01\rangle - a_1|10\rangle - a_2|11\rangle)_{24}
\tag{27}
$$

(with the probability $|ac|^2$) which is same as the state shown in (10). Now followed the procedure in Sect. 2, Bob can get the state $|\phi\rangle$ shown in (1). Similarly, Bob can recover $|\phi\rangle$ for other Alice's measurement results $\{|\psi_{00}\rangle_{13}, |\psi_{01}\rangle_{13}, |\psi_{11}\rangle_{13}\}$. The total successful probability of this new scheme is $4 \times |ac|^2 = 4|ac|^2$.

Of course, as far as the scheme in Sect. 3 is concerned, notice that instead of (16) the quantum channel $|\hat{\Psi}\rangle_{12} \otimes |\hat{\Psi}\rangle_{345}$ can be rewritten as

$$
\begin{aligned}
|\hat{\Psi}\rangle_{12} \otimes |\hat{\Psi}\rangle_{345} = {} & \frac{1}{2}|\hat{\psi}_{00}\rangle_{13}(a_1ac|000\rangle + a_2ad|011\rangle + a_3bc|100\rangle + a_4bd|111\rangle)_{245} \\
& + \frac{1}{2}|\hat{\psi}_{01}\rangle_{13}(a_2^*ac|000\rangle - a_1^*ad|011\rangle + a_4^*bc|100\rangle - a_3^*bd|111\rangle)_{245} \\
& + \frac{1}{2}|\hat{\psi}_{10}\rangle_{13}(b_1^*ac|000\rangle + b_2^*ad|011\rangle + b_3^*bc|100\rangle + b_4^*bd|111\rangle)_{245} \\
& + \frac{1}{2}|\hat{\psi}_{11}\rangle_{13}(c_1^*ac|000\rangle + c_2^*ad|011\rangle + c_3^*bc|100\rangle + c_4^*bd|111\rangle)_{245}.
\end{aligned}
\tag{28}
$$

Followed the scheme in Sect. 3 and the discussions for Bob above, Bob can obtain the state $|\phi\rangle$ shown in (1) with the successful probability $|ac|^2$.

5 Classical Communication Cost

The classical communication plays an important role in RSP. The classical information required in the RSP with three-party is divided into two parts. One is the classical information in step 1, i.e., Alice broadcasts the measurement outcome on particles $(1,3)$ to Bob and Charlie. The other is the classical information in step 2, assistant Charlie sends the measurement outcome on particle 5 to receiver Bob. Of course, all the classical channels used in this paper are broadcast channels.

Firstly, consider the RSP with maximally entangled states as the quantum channel. In Sect. 2, from (6) there are four possible outcomes for Alice with the equal probability $\frac{1}{4}$. Therefore, CCC for Alice is 2 cbits. As for Charlie, for each Alice's measurement result there are two possible outcomes on particle 5 with the same probability $\frac{1}{2}$ from (9). So, only 1 cbit is enough. The total CCC is 3 cbits. As far as the RSP in Sect. 3 is concerned, only 1.25 cbits are needed. In fact, since only Alice's measurement $|\hat{\psi}_{00}\rangle_{13}$ is useful for successful RSP, she can encode it by 0 and encode other Alice's measurement outcomes leading to failed RSP as 1. Thus, 1 cbit is enough for Alice. Moreover, the measurement result $|\hat{\psi}_{00}\rangle_{13}$ can be obtained with the probability $\frac{1}{4}$. For this measurement, there are two outcomes $\{|0\rangle_5, |1\rangle_5\}$ for Charlie with the equal probability. Thus only $1 \times \frac{1}{4} = 0.25$ cbit is consumed for Charlie.

Secondly, consider that the partially entangled joint system $|\hat{\Psi}\rangle_{12} \otimes |\hat{\Psi}\rangle_{345}$ is used as the quantum channel. On the one hand, if Alice obtains the maximally entangled quantum channel by the local filtering from Sect. 4, the RSP with three-party can be completed by following the procedures in Sect. 2 or Sect. 3. Otherwise, they cannot complete the RSP. In this case, additional classical communication is not required for Alice. So the total CCC is same as those schemes via the maximally entangled quantum channel in Sects. 2 and 3. On the other hand, if the local filtering is performed by the receiver after the sender and assistant's measurements from Sect. 4, for preparing the state $|\phi\rangle$ with real coefficients, from (23) there are four possible outcomes for Alice with the probability distribution

$$\begin{aligned}&\left\{\frac{1}{4}(|a_1ac|^2+|a_2ad|^2+|a_3bc|^2+|a_4bd|^2), \frac{1}{4}(|a_2ac|^2+|a_1ad|^2+|a_4bc|^2+|a_3bd|^2),\right.\\&\left.\frac{1}{4}(|a_3ac|^2+|a_4ad|^2+|a_1bc|^2+|a_2bd|^2), \frac{1}{4}(|a_4ac|^2+|a_3ad|^2+|a_2bc|^2+|a_1bd|^2)\right\}.\end{aligned} \tag{29}$$

Thus, from [28] the CCC of Alice is

$$\begin{aligned}c_1 = \frac{1}{4}\bigg[&(|a_1ac|^2+|a_2ad|^2+|a_3bc|^2+|a_4bd|^2)\log\frac{4}{|a_1ac|^2+|a_2ad|^2+|a_3bc|^2+|a_4bd|^2}\\&+(|a_2ac|^2+|a_1ad|^2+|a_4bc|^2+|a_3bd|^2)\log\frac{4}{|a_2ac|^2+|a_1ad|^2+|a_4bc|^2+|a_3bd|^2}\\&+(|a_3ac|^2+|a_4ad|^2+|a_1bc|^2+|a_2bd|^2)\log\frac{4}{|a_3ac|^2+|a_4ad|^2+|a_1bc|^2+|a_2bd|^2}\\&+(|a_4ac|^2+|a_3ad|^2+|a_2bc|^2+|a_1bd|^2)\log\frac{4}{|a_4ac|^2+|a_3ad|^2+|a_2bc|^2+|a_1bd|^2}\bigg],\end{aligned} \tag{30}$$

where $\log x$ is the logarithmic function with the base 2. As for Charlie, for each measurement result of Alice since there are two outcomes with the equal probability $\frac{1}{2}$, from (29) the CCC is

$$\begin{aligned} c_2 &= \bigg[\frac{1}{4}(|a_1ac|^2 + |a_2ad|^2 + |a_3bc|^2 + |a_4bd|^2) \\ &\quad + \frac{1}{4}(|a_2ac|^2 + |a_1ad|^2 + |a_4bc|^2 + |a_3bd|^2) \\ &\quad + \frac{1}{4}(|a_3ac|^2 + |a_4ad|^2 + |a_1bc|^2 + |a_2bd|^2) \\ &\quad + \frac{1}{4}(|a_4ac|^2 + |a_3ad|^2 + |a_2bc|^2 + |a_1bd|^2)\bigg] \times 1 \\ &= 1 \text{ cbit}. \end{aligned} \tag{31}$$

Therefore, the total CCC is $c_1 + c_2$ cbits.

Similarly, for preparing the state $|\phi\rangle$ with complex coefficients, see Sect. 3 and Sect. 4, from (28), since the only measurement result $|\hat{\psi}_{00}\rangle_{13}$ is useful for the successful preparation (see the discussions in Sect. 4), the CCC of Alice is 1 cbit, where 0 represents $|\hat{\psi}_{00}\rangle_{13}$ while 1 represents other results of Alice. At the same time, upon receiving the Alice's measurement $|\hat{\psi}_{00}\rangle_{13}$ with the probability $\frac{1}{4}(|a_1ac|^2 + |a_2ad|^2 + |a_3bc|^2 + |a_4bd|^2)$, Charlie has two outcomes with the equal probability $\frac{1}{2}$ for the successful preparation. Thus, only $\frac{1}{4}(|a_1ac|^2 + |a_2ad|^2 + |a_3bc|^2 + |a_4bd|^2) \times 1 = \frac{1}{4}(|a_1ac|^2 + |a_2ad|^2 + |a_3bc|^2 + |a_4bd|^2)$ cbits is enough for Charlie. So, the total CCC is $1 + \frac{1}{4}(|a_1ac|^2 + |a_2ad|^2 + |a_3bc|^2 + |a_4bd|^2)$ cbits.

6 Discussions and Conclusions

In Sect. 2, with the assumption that Bob has particle 2, we have presented one scheme to remotely prepare an arbitrary two qubit state to Bob. Similarly, Alice can prepare this qubit to Charlie if particle 2 belongs to Charlie. Our scheme can also be viewed as from other points. The first one is similar to the quantum state sharing scheme of teleportation [18–22]. In detail, after the sender Alice measures on his own particles, two receivers share one state such as $(a_1|000\rangle + a_2|011\rangle + a_3|100\rangle + a_4|111\rangle)_{245}$ which includes the secret state (the prepared state). Each receiver Bob or Charlie can not recover the prepared state from the collapsed joint system without another receiver's help. So after receiving the measurement outcome of Alice, to complete RSP one receiver has to agree to help the other. The second one is that one receiver can be viewed as a controller. Without the controller's measurement on its particle, the receiver can not obtain the prepared state. In this case, it is named with the controlled RSP or assisted RSP [14, 23]. Also, it can be realized as follows: the controller performs the measurement on its particle under the basis $\{|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\}$ firstly. Then Alice makes the generalized measurement on his particles under our constructed basis. This new scheme is similar to the procedure that one sender remotely prepares an arbitrary two qubit state to Bob via two EPR pairs as the quantum channel [27]. The total CCC is same as the present one in Sect. 2 for the broadcast channel. For the non-broadcast channel, this new scheme saves 2 cbits (Alice to the controller).

In conclusion, some quantum schemes are proposed to remotely prepare an arbitrary two-qubit from a sender to either of two receivers via the maximally entangled or the partially entangled quantum channel. Such an arbitrary two-qubit includes two cases, i.e., an arbitrary two-qubit with real coefficients and complex coefficients. To design these schemes, we have constructed some general and useful unitary matrices and operations for each participant. For the maximally entangled quantum channel, it is shown that the receiver can recover the prepared states with real coefficients and complex coefficients in the probabilities 100% and 25%, respectively. And these schemes consume 3 cbits and 1.25 cbits per arbitrary two-qubit with real coefficients and complex coefficients, respectively. When the partially entangled states are used as the quantum channel, the RSP schemes possess $4|ac|^2$ and $|ac|^2$ successful probabilities per arbitrary two-qubit with real coefficients and complex coefficients, respectively. Moreover, the total CCC is same as those schemes via the maximally entangled quantum channel if the local filtering has been completed before the measurement of sender.

Comparing with the previous RSP schemes of two-party [2–11], the present schemes have the following features. First, our protocols have investigated the RSP of an arbitrary two-qubit state from a sender to either of two receivers. Second, two kinds of quantum channel, maximally entangled states and non-maximally entangled states, have been used in this paper. However, in previous schemes, only one of them has been discussed for each paper. Third, the total CCC have been evaluated. The total CCC includes two parts, while the earlier RSP schemes have considered the CCC from one sender to one receiver.

Moreover, comparing with the previous RSP schemes of three-party [13, 14] in which two assistants share the knowledge of the prepared state (coefficients), only one helper in the present RSP schemes knows the prepared state completely while the other assistant has no knowledge of the prepared state. It shows that the roles of two helpers in present schemes are different from them in [13, 14]. At the same time, the prepared state and quantum resources are also different. Besides, by constructing some general and useful measurement bases, we have discussed the RSP of an arbitrary two-qubit via various quantum channels while only partially entangled two-qubit state via the non-maximally entangled quantum channel has been discussed in [16]. It means that the result in [16] is only a special case of our schemes.

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